

## DEMYSTIFYING HOLDING PERIOD RETURN: A BEGINNER-FRIENDLY GUIDE

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**Abstract:** The concept of holding period return ( $R$ ) is a fundamental measure in finance, representing the ratio of future proceeds to the initial investment. For bonds, this calculation is defined as  $R = (B_1 - B_0 + iF)/B_0$ , where  $B_t$  denotes the bond valuations at time  $t$ ,  $iF$  represents interest payments at the interest rate  $i$  on face value  $F$ , and  $M$  signifies maturity, discounted at rate  $k$ , known as the yield to maturity. Corporate bonds often entail semi-annual interest payments, equivalent to half the annual  $iF$  amount. These interest payments can be conceptualized as an annuity,  $iF/k(1 - 1/[1+k]^M)$ , while the face value is  $F/(1+k)^M$ . This abstract delves into the mathematical intricacies of holding period returns for bonds and provides insights into their underlying principles.

**Keywords:** Holding Period Return, Bond Valuation, Yield to Maturity, Corporate Bonds, Interest Payments

### Introduction

A holding period return  $R$  is a ratio of future proceeds divided by its initial investment. For a bond it is  $R = (B_1 - B_0 + iF)/B_0$  with bond valuations  $B_t$  at time  $t$  with interest payments of  $iF$  at interest rate  $i$  on face value  $F$ , and with a maturity  $M$  discounted at rate  $k$  which for bonds is called the yield to maturity. Interest payments on corporate bonds are often paid twice a year as half the annual amount of  $iF$ . The interest payments are an annuity as  $iF/k(1 - 1/[1+k]^M)$  and the face value is  $F/(1+k)^M$  or:

$B_0 = iF/k(1 - 1/[1+k]^M) + F/(1+k)^M$  and  
 $B_1 = iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1}$ .

Thus:

$R = \{iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1} - iF/k(1 - 1/[1+k]^M) - F/(1+k)^M + iF\} / \{iF/k(1 - 1/[1+k]^M) + F/(1+k)^M\}$ , canceling  $F$ :

$R = \{i/k(1 - 1/[1+k]^{M-1}) + 1/(1+k)^{M-1} - i/k(1 - 1/[1+k]^M) - 1/(1+k)^M + i\} / \{i/k(1 - 1/[1+k]^M) + 1/(1+k)^M\}$ , expanding the annuities:

$R = \{i/k - [i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} - i/k + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\}$ , canceling like terms:

$R = \{-[i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\}$ , multiplying by  $(1+k)^M$ :

$R = [-i(1+k)/k + (1+k) + i/k - 1 + i(1+k)^M] / [i(1+k)^M/k - i/k + 1]$ , and multiplying by  $k$ :

$R = [-i(1+k) + (1+k)k + i - k + ik(1+k)^M] / [i(1+k)^M - i + k]$ , and expanding:

$R = [-i - ik + k + kk + i - k + ik(1+k)^M] / [i(1+k)^M - i + k]$ , and canceling:

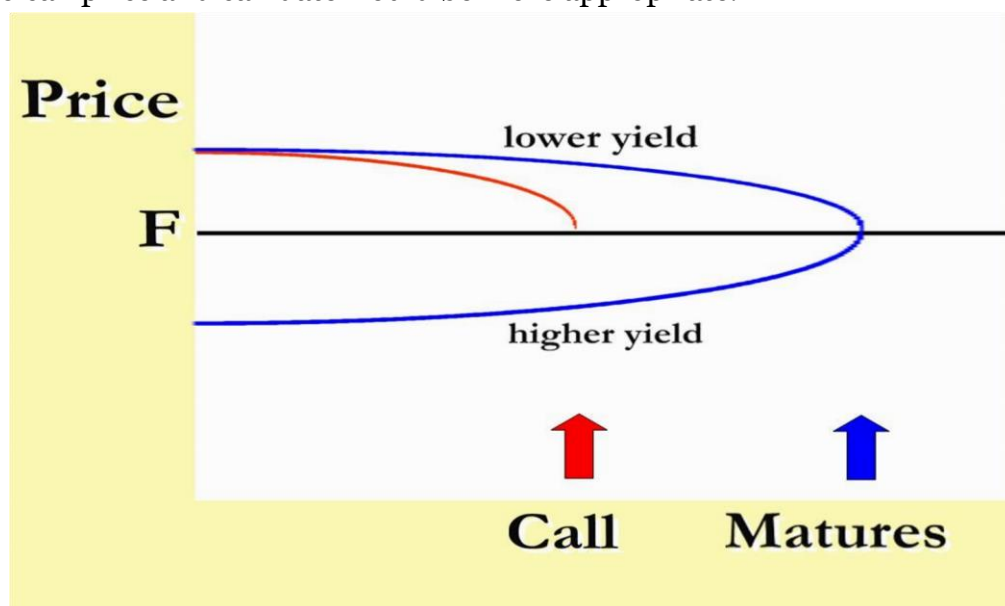
$R = [-ik + kk + ik(1+k)^M] / [i(1+k)^M - i + k]$ .

The numerator is a multiple of the denominator by  $k$ , therefore  $R = k$ .

Here are some different examples of holding period returns for various bonds discounted at a 10 percent yield: Coupon

Interest	Maturity	Price	Maturity	Price	Yield
0	2	86.64	1	90.91	10
1	3	77.62	2	84.38	10
5	30	52.87	29	53.15	10
10	20	100.00	19	100.00	10
12	5	107.58	4	106.34	10
10			15	20	142.56
			19		141.82

Do be wary of bonds trading at a premium and/or convertible bonds. For bonds trading at a premium, yield to call price and call date would be more appropriate.



For a common stock the holding period return  $R$  with valuations  $P_t$  and  $D_t$  at time  $t$ , the yearly return would be  $R = (P_1 - P_0 + D_1)/P_0$ . Often dividends are paid four times a year with an associated decrease in value on its exdividend date; exchange traded funds often pay monthly. A common stock with a dividend payment of  $D_t$  and discounted at rate  $k$  with growth rate  $g$ , is valued as:

$$P_0 = D_1/(k-g) = D_0(1+g)/(k-g);$$

Substituting:  $R = (D_1[1+g]/[k-g] - D_1/[k-g] + D_1)/(D_1/[k-g])$ , Canceling  $D_1$ :

$R = ([1+g]/[k-g] - 1/[k-g] + 1)/(1/[k-g])$ , Multiplying by  $(k-g)$ :  $R = (1 + g - 1 + k - g)$  Which results in  $R = k$ .

Here are some examples of different common stocks with different dividends and different growth rates discounted at a 10 percent discount rate:

Growth Rate	Current Dividend	Current Price	Next Dividend	Next Price	Yield
0	2.00	20.00	2.00	20.00	10
4	1.00	17.33	1.04	18.03	10
6	4.00	106.00	4.24	112.36	10
8	1.00	54.00	1.08	58.32	10

Do be wary that tax considerations were neutral here whereas in reality capital gains tax rates and the tax rates upon dividends may differ.

Note that  $D_1 = E_1(1-b)$  where  $E$  is a firm's earnings,  $b$  is the firm's retention rate, and a firm's endogenous growth rate  $g$  may be determined by  $g = br$  where  $r$  is the firm's rate of return. Earnings are achieved on the firm's assets  $A$  or  $E_1 = A_0r$ .

Thus:

$$P_0 = D_1/(k-g) = E_1(1-b)/(k-g) = A_0r(1-b)/(k-br).$$

Likewise  $k = D_1/P + g = E_1(1-b)/P + br = A_0r(1-b)/P + br$ . Where  $r$  is greater (less) than  $k$ ,  $P$  will be valued greater (less) than  $A$ . However, when  $r$  is greater than  $k$ , a lesser dividend and a greater retention rate resulting in increased growth will increase valuations, whereas when  $r$  is less than  $k$  then a greater dividend and lesser retention and lower growth rates will increase valuations albeit still below asset valuation  $A$ . Consider  $A_0$  equaling 100, with  $r$  equaling .12 and  $E_1$  equaling 12, the valuations are:

b	1-b	g	$D_1$	$k=.14$	$k=.12$	$k=.10$
0	1	.00	12	85.7	100.0	120.0
1/4	3/4	.03	9	81.8	100.0	128.6
1/2	1/2	.06	6	75.0	100.0	150.0
3/4	1/4	.09	3	60.0	100.0	300.0

When  $r$  equals  $k$ ,  $P$  equals  $A$  validating the Miller and Modigliani proposition that dividends do not matter. A shortcut to a payout ratio is dividend yield times the P-E ratio or  $(D/P)(P/E) = D/E = (1-b)$ . A valuation using the price-earnings ratio follows from  $P_0 = E_1(1-b)/(k-br)$  where  $P-E = (1-b)/(k-br)$  and when multiplied by the expected earnings  $E_1$  provides a stock valuation. In equilibrium when  $k$  equals  $r$ , the P-E ratio equals  $1/k$ .

## Conclusion

While obvious once the mathematics are examined, I repeatedly ask my students what is the holding period return to bonds and stocks and rarely get a correct response. Thus I'm repeatedly reminded that this exercise is well worth the review.

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