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UNVEILING THE CONSTRAINING ATTRIBUTES OF LOCAL AGGREGATE QUANTILE REGRESSION IN DISSEMINATION MODELS

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Abstract: *Based on the research and analysis of the development status and characteristics of the Hydrogen fuel cell vehicle industry in Jinhua, combined with the domestic and foreign technology development direction and industry development trend, through extensive demand research and the construction of a standard system, this paper guides the technological innovation and standard creation of Hydrogen fuel cell vehicles, and promotes the sustainable, healthy, scientific and orderly development of the industry. In the process of building the Hydrogen fuel cell vehicle standard system, we should do a good job in top-level design, build a good standard system framework, and promote the better development of the entire standard system. After the introduction of various standards, we should popularize and implement them.*

Keywords: *Composite quantile regression, parameter estimation, diffusion models, option pricing, interest rate term structure.*

1. Introduction

Composite quantile regression (CQR) is proposed by Zou and Yuan (2008) for estimating regression coefficients in classical linear regression models. More recently, Kai el.(2010) considers a general non-parametric regression models by using CQR method. However, to our knowledge, little literature has researched parameter estimation by CQR in diffusion models. This motivates us to consider estimating regression coefficients under the framework of diffusion models. In this paper, we consider the diffusion model on a filtered probability space (□,F,(Ft)t□0,P)

(1.1) dX_t □□(t)b(X_t)dt □□(X_t)dW_t ,

□(t) W^t is the standard Brownian motion. b(□) and□(□) are known

where is a time-dependent drift function and functions. Model (1.1) includes many famous option pricing models and interest rate term structure models, such as Black and Scholes(1973), Vasicek(1977), Ho and Lee(1986), Black, Derman and Toy (1990) and so on.

□(t)

We allow being smooth in time. The techniques that we employ here are based on local linear fitting (see Fan and Gijbels(1996)) for the time-dependent parameter. The rest of this paper is organized as follows. In Section 2, we propose the local linear composite quantile regression estimation for the drift parameter and study its asymptotic properties. The asymptotic relative efficiency of the local estimation with respect to local least squares estimation is discussed in Section 3. The proof of result is given in Section 4.

2. Local estimation of the time-dependent parameter

{X^{t_i},i □ 1,2,□,n □1} t^1 □ t^2 □□□ $t^{n□1}$. Denote

Let the data be equally sampled at discrete time points,

Yti □ *Xti*□1 □ *X ti* ,□*ti* □*Wti*□1 □*Wti* , and □*i* □ *ti*□1 □*ti*.Due to the independent increment property of Brownian motion
Wt ,□*ti* are independent and normally distributed with mean zero and variance1*i*.. Thus, the discretized version of the model (1.1) can be expressed as
(2.1) *Yti* □ □(*ti*)*b*(*X ti*)□*i* □□(*X ti*)□*iZti* ,
Zti 1/ □ *i* .The first-order discretized
where are independent and normally distributed with mean zero and variance approximation error to the continuous-time model is extremely small according to the findings in Stanton (1997) and Fan and Zhang(2003), this simplifies the estimation procedure.
Suppose the drift parameter □(*t*) to be twice continuously differentiable in *t* . We can take □(*t*) to be local
t o , we use the approximation linear fitting. That is, for a given time point
(2.2) □(*t*) □□(*t*o) □□'(*t*o)(*t* □*t*o)
for*t* in a small neighborhood of *t*o . Let *h* denote the size of the neighborhood and *K*(□) be a nonnegative weighted function. *h* and *K*(□) are the bandwidth parameter and kernel function, respectively. Denoting □o =□(*t*o) and
□1 □□'(*t*o) , (2.2) can be expressed as
(2.3) □(*t*) □□o □□1(*t* □*t*o) .
□(*t*)
Now we propose the local linear CQR estimation of the drift parameter . Let
k
□*k* =___
□□*k* (*r*) □□*kr* □□*I*{*r*□o},*k* □ 1,2,□,*q* ,which are *q* check loss functions at *q* quantile positions: *q* □ 1 . Thus,
□(*t*)
following the local CQR technique, can be estimated via minimizing the locally weighted CQR loss

q *n* *Y ti* □1
(2.4) □ {□□□*k*{ [*b*(*Xti*)] □□o*k* □□1(*ti* □*t*o)}*Kh*(*ti* □*t*o)}
ti □*t* o) *Kh* (*ti* □*t*o)=*K*(where *h* and *h* is a properly selected bandwidth. Denote the minimizer of the locally weighted
(□ˆo1,□ˆo2,□,□ˆo*q*,□ˆ1)*T*
CQR loss (2.4) by . Then, we let

q
(2.5) □ˆ(*t*o) □ 1 □□ˆ o*k*

q *k*□1

We refer to □ˆ(*t*o) as the local linear CQR estimation of□(*t*o) , for a given time point *t* o . To obtain the
□ˆ(□)

---

[1] *k*□1 *i*□1 *i* ,

estimated function , we usually evaluate the estimations at hundreds of grid points.
In order to discuss the asymptotic properties of the estimation, we introduce the following assumptions. Throughout this paper, M denotes a positive generic constant independent of all other variables.

$b(□)$ $□(□)$
(A1) The functions and in model (1.1) are continuous.

$K(□)$
(A2) The kernel function is a symmetric and Lipschitz continuous function with finite support $[□M, M]$.

$h = h(n) □ 0$ $nh □ 0$.
(A3) The bandwidth and

$F(□)$ $f(□)$
Let and be the cumulative density function and probability density function of the error, $g(□)$ $[a,b]$ respectively. denotes the density function of time, usually a uniform distribution on time interval . Define
$□_j □ □ u^j K(u)du,\ □_j □ □ u^j K^2(u)du,\quad j □ 1,2,□$

and

$$(2.6)\quad R(q) □ \frac{1}{q^2} □ □ \frac{□_{kk'}}{f(c_k) f(c_{k'})},\quad k □ 1,\ k' □ 1,\ q$$

$c_k □ F^{□1}(□_k)$ and $□_{kk'} = □_k □ □_{k'} □ □ □\ k \quad k'$. where

$□\hat{}(t_0)$
Theorem 2.1 Under assumptions (A1)-(A3), for a given time point t_0, the local CQR estimation from (2.5) satisfies,

$$(2.7)\quad E[□\hat{}(t_0)] □ □(t_0) □ \frac{1}{2} □''(t_0) □_2 h^2 □ o(h^2)$$

$$(2.8)\quad Var[□\hat{}(t_0)] □ \frac{1}{nh} \frac{□_0}{g(t_0) b^2(X_{t_0})} □ □^0 (X^t)\ R(q) □ o\left(\frac{1}{nh}\right)$$

and, as $n □ □$,
2

(2.9) $nh\{□\hat{}(t_0) □□(t_0) □ ^1 □''(t_0)□h^2\} □_L N(0, \frac{\quad _0}{2} □□^0$ (X^t) $R(q))$

2 $g(t_0)b(X_{t0})$

$□^L$ means convergence in distribution.

Where

3. Asymptotic relative efficiency

We discuss the asymptotic relative efficiency(ARE) of the local linear CQR estimation with respect to the local linear least squares estimation(see Fan and Gijbels(1996)) by comparing their mean-squared errors(MSE).From

$□\hat{}(t^0)$. That is, theorem 2.1, we obtain the MSE

2

(3.1) $MSE[□\hat{}(t_0)] □ [^1 □''(t_0)□_2]^2 □ ^1 \frac{\quad _0}{2} □□^0$ (X^t) $R(q)$ $□o(h^4 □ ^1)$

2 $nh\, g(t_0)b(X_{t0})$ nh

We obtain the optimal bandwidth via minimizing the MSE (3.1), denoted by

$h_{opt}(t_0)] □ [□□_0 2(X_{t0})R(q) ^{-}] \bar{1}5 n^{□}5\ 1$

2 2

$g(t_0)b(X_{t0})[□''(t_0)□_2]$

.

$□(t^0)$, denoted by $□\hat{}^{LS}(t^0)$, is The MSE of the local linear least squares estimation of

2

(3.2) $MSE[□\hat{}_{LS}(t_0)] □ [^1 □''(t_0)□_2]^2 h^4 □ ^1 \frac{\quad _0}{2} □□^0$ (X^t) $□o(h^4 □ ^1)$

2 $nh\, g(t_0)b(X_{t0})$ nh

and the optimal bandwidth is

$□□2(X)$ 1 $\frac{1 \quad 0 \quad t_0}{}$ $\bar{5}$ $□5$

opt

$h_{LS}(t_0)] □ [$ 2 2 $] n$

$g(t_0)b(X_{t0})[□''(t_0)□_2]$

.

By straightforward calculations, we have, as $n □□$,

$MSE[□\hat{}_{LS}(t_0)] □ [R'(q)]^{□\frac{4}{5}}$

$MSE[□\hat{}(t_0)]$

Thus, the ARE of the local linear CQR estimation with respect to the local linear least squares estimation is

□ $\frac{4}{5}$

(3.3) $ARE(□\hat{}(t_0),□\hat{}_{LS}(t_0))$ □ $[R(q)]$

(3.3) reveals that the ARE depends only on the error distribution. The ARE we obtained is equal to that in Kai el.(2010).

$ARE(□\hat{}(t^o),□\hat{}^{LS}(t^o))$ for some commonly seen error distributions. Table 1 in Kai Table 3.1 displays el.(2010) can be seen as ARE for more error distributions.

Table 3.1: Comparisons of $ARE(□\hat{}(t0),□\hat{}LS(t0))$ **for the values of** q

Error	q □1 q □ 5	q □ 9 q □ 19	q □ 99
$N(0,1)$	0.6968 0.9339	0.96590.9858	0.9980
Laplace	1.7411 1.2199	1.1548 1.0960	1.0296
$0.9N(0,1)$ □$0.1N(0,10^2)$	4.0505 4.9128	4.70693.5444	1.1379

From Table 3.1, we can see that the local linear CQR estimation is more efficient than the local linear least squares estimation when the error distribution is not standard normal distribution. When the error distribution is

$N(0,1)$ and q □ 1,5,9,19,99 , the $ARE(□\hat{}(t0),□\hat{}LS(t0))$ is very close to 1, which demonstrates that the local linear

CQR estimation performs well when the error conforms to the standard normal distribution too.

4. Proof of result

□$S11$ S 12 □

S □□ □

In order to prove theorem 2.1, we first give some notations and lemmas. Let □$S21$ S 22□ , and

□□11 □ 12 □

□□□ □

□□21 □ 22□ , where $S11$ is a q□q diagonal matrix with diagonal elements $f(ck),k$ □1,2,□,q ,

q

S □□ $f(c)$

$S12$ □ (□1 $f(c1)$,□1 $f(c2)$,□,□1 $f(cq)$)T , $S21$ □ $S12T$ and 22 2□k□1 k . □ 11 is a q□q matrix with (k,k') -

□□0 k,k',k,k' □ 1,2,□,q, □12 □ (□1□kq'□1□$1k'$,□1□qk'□1□$2k'$,□,□1□kq'□1□qk')T

□21=□12 ' □22 □□2□kq,k'□1□kk'

element , and .

□□ □(X) □□

k $\sqrt{\ }$ 0k 0 k $\sqrt{\ }$ 1 0 u □ nh □□ □□(t)□ to c

Furthermore, let □□ $b(X_t0)$ □□ , □,v □ h nh□□ □□' (t) □

1 □ ti □to □ di, k □i,k □ □uk □ ck □ □ □□r i
□ v □ □□b(Xti) b(Xto) □□ with ri □ □(ti) □□(to) □□' (to
and nh □ h □ .Write)(ti □to) .
□□ □(Xti) □(Xto)□ □
Define i,k to be ti k i,k titi k . Let n 11 12
1q 1(q□1) with
□ 1 □n □ □,q w1(□ q□1) □ 1 □□q n □i□,kKh(ti □to) ti □t o
w1k □ □i,k Kh(ti □to),k □1,2,
nh i□1 , and nh k□1 i□1 h .

Lemma 4.1 Under assumption (A1)-(A3), minimizing (2.4) is equivalent to minimizing the following term:

q □ n □i*,k Kh(ti □to)□ q n □i*,kKh(ti □to)(ti □to) q
Ln(□) □ □uk □□□□v□□□□Bn,k (□)
k□1 □ i□1□ k□1 i□1k□1

□□ I □Z □c □d
□ □ □ □ □ T
b(X)□(X)□□ □ W □ (w ,w ,□,w ,w /)
□ 1/2 □T Sn□□ （Wn*）T □□ op (1)

□=(u ,u ,□,u ,□)
1 1 q with respect to , where

□
Bn,k □□□□□i□n1 □□□□Kh □ti □to □□i,1 □□□□I□□ ti k d□ i,1b□Xti ti □
zb□Xttii □□□ - I□□ ti k di,1b□Xti ti □□□□□□□dz□ □□□ Sn □□□□SSnn,,1121
SSnn,,1222 □ □□□,
0 □ □Z □c □

□ □□ □X □ □□□X □□□□ □□□Z □c □ □□X □ □□□□□ □, □

□ n b□Xti □ □
Sn,11 □□□Kh □ti □to □ □ i □□S11 with □□ i□1 nh□ X t □□ , Sn,21 □SnT ,12 ,

Sn,12 □□□□n Kh □ti □to □ti □to b□X □ti □i □□□□f □c1 □,f □c2 □,□,f □cq □□T

□□ i□1 h nh□ X t □□,

□q □ck □□n □□Kh □ti □to □(ti □2to)2 b□X□ti □i □□ □

1 □□X t □

√nh h√nh

Sn,22 □ f

and □ h nh□ Xt □ □.
k□1 i□1 □

The proof of lemma 4.1 is similar to lemma 2 and lemma 3 in Kai el.(2010).

Proof of theorem 2.1

Using the results of Parzen(1962), we have

1 n □ti □to□ j
□Kh □ti □to□ j □P g□to□u j nh i□1 h

□ P means convergence in probability. Thus,
where

g□t0□b□Xto □ g□t0□b□Xto S12 □
□□S11 □ S
Sn □P S □ □ 22□
□□X to □ □□Xto □ □S21 .

According to lemma 4.1, we have
L□□□□□ 1g□t1 □ b□X to □□T S□□□Wn* □T□□ op □1□
Ln□□□□□Wn*□T□ converges in probability to the convex function Since the convex function
1 g□to □b□X to □ T
□ S□

2 □□X t □
o , according to the convexity lemma in Pollard(1991), for any compact set, the quadratic
L□□□□ □
approximation to holds uniformly for . Thus, we have

ˆ □□ g□to □b□X to □ S□1Wn* □o p □1□
□n

□□X t □
o .

Define □i,k □ I□zti □ck □ □□k and Wn □ □w11,w12,□w1q,w1□q□1□□T
with
w1k □ 1 □n □i,kKh □ti □to□,k □1,2,□,q w1□q□1□ □ 1 □q □n □i,k Kh □ti □to □ ti □to
nh i□1 , and nh k□1 i□1 h .

By using the central limit theorem and the Cramer-Wald theorem, we have

[1] .

W^n □ $E(W^n)$ $N(0,I)$
(4.1) □ √
L (q□1)□(q□1) $Var(W_n)$
.

Notice that Cov(□$_{i,k}$,□i,k') □□kk' andCov(□$_{i,k}$,□j,k') □ 0 Ifi □ j . We have

1 n 2 (ti □t0) j
□Kh (ti □t0) j □P $g(t_0)v_j$. nh i□1 h

$Var(W)$ □ $g(t)$□. W □ $N(0,g(t)$□)

Thus, $^{n\,0}$. Combining the result (4.1), we have $^{n\,L\,0}$. Moreover, we have
1 n
* ^{2} *
$Var(w1k$ □$w1k$) □ □Kh (ti □t0)Var(□i,k □□i,k)
nh i□1

1 n 2 | $d_{i,k}$ | $b(Xti)$
□ □K_h (t_i □t_0)[$F(c_k$□) □$F(c_k)$] □□$_p$ (1) nh i□1 □(Xti)

And

n q
* 1 2 ti □t 0 *
$Var(w1(q$□1) □ $w1(q$□1)) □ □Kh (ti □t0) Var(□□i,k □□i,k)

nh i□1 h k□1

$q2$ n ^{2} t_i□t $_0$| $d_{i,k}$ | $b(Xti)$
□ □K_h (t_i □t_0) max$_k$[$F(c_k$□) □$F(c_k)$] □□$_p$ (1). nh i□1h □(Xti)

$Var(w^{n*}$ □ w^n) □□ ^{p} *
Therefore, (1) . Using Slutsky's theorem yields w_n □$_L$ N (0, $g(t_0)$□).

Thus,
□ □(Xt0) □1 * □2 (Xt) □1 □1

□n □ S E(Wn) □L N(0, 0 2 S □S) g(t0)b(Xt) g(t0)b (Xt)

0 0

So the asymptotic bias of □ˆ(tO) is:

bias(□ˆ(tO)) □ 1 □(Xt0) □q ck □ 1 □(Xt0) eqT□1(S11)□1E(W1 * n) q b(Xt0) k□1 q nh g(t0)b(Xt0)

□ 1 □(Xt0) □q ck □ 1 □(Xt0) □n Ki □q 1 □□F(ck □ di,kb(Xt0)) □ F(ck)□□ , q b(Xt0) k□1 q nh g(t0)b(X t0) i□1 k□1 f (ck) □□ □(X ti) □□ where

Ki □ Kh(ti □t0),eq□1 □ (1,1,□, 1)T and W1*n □(w11* ,w12* ,...w1*q)T .

q
c □ 0
z □ k , and
Note that ti is symmetric, thus k□1

1 q 1 □ di,kb(Xti) □ rbi (Xti)
□ □F(ck □) □ F(ck)□□□ (1□oP(1)). q k□1 f (ck) □□ □(Xt) □□ □(Xt)

i i

Therefore,

1 □(Xt0) n rbi (X ti)
bias(□ˆ(t0)) □ □Ki (1□oP (1)). Since
nh g(t0)b(X t0) i□1 □(X ti)
n rb(X) g(t)□" (t)b(X)
1
□Ki i ti □ 0 0 t0 □2 h2 (1□oP (1)). We have
nh i□1 □(X ti) 2□(X t0)

1

2

bias(□ˆ(*t*₀)) □□"(*t*₀)□₂*h*² □*o*P(*h*²). and

□²(*X*)

Var[□ˆ(*t*0)] □ 1 2 *t*0 12 *eqT*□1(*S*□1□*S*□1)11*eq*□1 □*op*(1) *nh g*(*t*₀)*b* (*X t*0) *q nh*

□ 1 ___ 0 2 *v*0□2 (*X t*) *R*(*q*) □*o*p(1). *nh g*(*t*₀)*b* (*X t*0) *nh*

This completes the proof.

References

Black, F., Derman, E. and Toy, W.(1990).*A one-factor model of interest rate and its application to treasury bond options*. Finan. Analysts' J., **46:**33-39.

Black,F. and Scholes, M. (1973). *The pricing of options and corporate liabilities*. J. Polit. Economy, **81:**637-654.

Fan, J. and Gijbels, I.(1996).*Local polynomial modelling and its applications*. Chapman and Hall, London.

Fan, J., Jiang, J., Zhang, C. and Zhou, Z.(2003).*Time-dependent diffusion models for term structure dynamics and the stock price volatility*. Statistica Sinica, **13:**965-992.

Kai,B.,Li,R. and Zou, H.(2010).*Local composite quantile regression smoothing: an efficient and safe alternative to local polynomial regression*.J.R. Statist. Soc. B,**72:**49-69.

Parzen, E.(1962).*On estimation of a probability density function and model*. Ann. Math. Statist., **33:**1065-1076.

Pollard, D.(1991).*A symptotics for least absolute deviation regression estimations}.Econometr*.Theory,**7:**186-199.

Stanton,R.(1997).*A nonparametric model of term structure dynamics and the market price of interest rate risk*. J. Finance, **52:**19732002.

Vasicek,O.A.(1977).*An equilibrium characterization of the term structure*. J. Finan. Econom., **5:**177-188.

Zou,H. and Yuan, M.(2008).*Composite quantile regression and the oracle model selection theory*.Ann. Statist.,**36:**1108-1126. T.A.Louis, *Finding observed information using the EMl gorithm*, J.Royal Stat.Soc, 1982, B44: 226-233.